Money and Endogenous Growth

1. INTRODUCTION

Tobin (1965) argues that money is not superneutral because an increase in the anticipated growth of the nominal money supply reduces the real interest rate and increases the long-run capital-output ratio. The rationale for the Tobin effect has been denied by Sidrauski (1967) within the context of an infinite-horizon, representative-agent model with an inelastic supply of labor. Fischer (1979) and Asako (1983) show that, as long as preferences are nonseparable, non-neutralities may occur during the transition path. There is a close connection between Sidrauski's superneutrality result and Ricardian debt neutrality. Weil (1986), Marini and van der Ploeg (1988) and van der Ploeg (1991) show that when agents have no operational intergenerational bequest motive and there is entry of new generations, increases in monetary growth accompanied by lump-sum transfers have real

The authors have benefitted from helpful comments of Olivier Blanchard, Willem Buiter, and two anonymous referees on an earlier draft. They acknowledge support from the CEPR research program in International Macroeconomics, financed by grants from the Ford Foundation (no. 890-0404) and the Alfred P. Sloan Foundation (no. 88-4-23). This paper is also supported by a SPES grant from the Commission of European Communities (no. 0016-NL (A)).

1. Dornbusch and Frankel (1973) and Fischer (1974) show that superneutrality no longer holds when money is a substitute for capital in production. Brock (1974) obtains departures from superneutrality by putting leisure in a nonseparable fashion into the utility function. Stockman (1981) demonstrates that an inverse Tobin effect may arise when money is needed for buying capital goods. Wang and Yip (1992) allow for endogenous labor and compare the conditions for which money is non-neutral under the money-in-the-utility-function, the cash-in-advance, and the transactions-costs approaches.

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Journal of Money, Credit, and Banking, Vol. 26, No. 4 (November 1994) Copyright 1994 by The Ohio State University Press
effects. In this literature money only affects growth in the short run, because in steady state growth is given by the sum of population growth and an exogenous rate of technical progress. This means that inflation is in the long run entirely a monetary phenomenon. In the short run economic growth is higher when the capital-output ratio is lower and thus the marginal productivity of capital is higher than in steady state as then households have a greater incentive to postpone consumption, but in the long run the growth rate is exogenous. These results on money and growth arise naturally in the neoclassical theory of economic growth developed by Solow (1956), Swan (1956) and Cass (1965). This paper, in contrast, addresses the issues of money and growth within the context of the new theories of endogenous growth developed by Romer (1986) and Lucas (1988).

The new theories of growth allow for an endogenous long-run growth rate by assuming that production is proportional to a broad measure of the capital stock. The capital stock thus includes the physical stock of machines, the stock of ideas or knowledge both of the individual firm and of its competitors, and the stock of infrastructural public goods. Production externalities, arising from learning by doing, R&D, and infrastructural public goods now affect the marginal productivity of capital, the real interest rate, and thus growth. However, macroeconomic fiscal and monetary policies do not affect growth. The reason is that the new theories of endogenous growth are based on Ramsey (1928) which assumes dynasties with an operative intergenerational bequest motive and thus implies Ricardian debt neutrality. This is an unnecessarily restrictive framework for addressing questions about the effectiveness of macroeconomic policy as then money is superneutral. However, in Alogoskoufis and van der Ploeg (1990, 1991) we show that once the theories of endogenous growth are extended to allow for overlapping generations of households, demand-side policies (even when accompanied by changes in lump-sum taxes and subsidies) have real effects. This extension introduces consumption externalities arising from missing markets, as current generations cannot trade with future generations. An increase in government debt arising from an intertemporal shift in taxation (or an increase in government consumption) then reduces the growth rate. The mechanism is that future taxes are partially shouldered by future generations so that private consumption rises (is not fully crowded out), fewer resources are available for private saving and investment, and thus the growth rate falls.

This paper provides a coherent framework of endogenous growth and overlapping generations with money in the utility function and inelastic labor supply in which monetary growth permanently affects real growth. An increase in monetary growth

2. Aiyagari and Gertler (1985) use the more conventional overlapping generations model of Diamond (1965) to discuss the validity of various monetarist propositions under alternative budgetary rules. Of course, money is also non-neutral if changes in monetary growth are accompanied by changes in distortionary taxes.


4. After writing this paper we became aware of the work of Mino and Shibata (1992), who also investigate the effects of monetary and fiscal policies within the context of a model of endogenous growth and overlapping generations with money in the utility function.
then no longer leads to an identical increase in inflation, and also money is no longer the sole determinant of inflation in the long run. We also show that increases in public debt and public consumption damage growth prospects and thus increase inflation even when accompanied by increases in *lump-sum* taxes and a constant rate of growth of the nominal money supply.\textsuperscript{5}

This paper provides an alternative to recent work on money and endogenous growth within the context of the cash-in-advance approach. Blackburn and Hung (1991), Jones and Manuelli (1991), and Mino (1991) show that, as long as money is needed to purchase investment as well as consumption goods, it is not necessary to move away from infinite horizons and debt neutrality in order for monetary policy to be non-neutral.\textsuperscript{6} In such a framework monetary growth and inflation can act like a tax on investment and thus depress growth.\textsuperscript{7} Within a framework where transactions costs explain the demand for money and transaction time affects human capital formation and endogenous growth, Wang and Yip (1992) show that an increase in monetary growth raises transaction time and reduces real growth. This paper, in contrast, provides a channel through which monetary growth raises real growth.

Section 2 formulates a monetary model of endogenous growth and noninterconnected overlapping generations. Section 3 demonstrates that, if there is no new entry of generations, debt neutrality holds and thus monetary growth, government consumption, and government debt do not affect real growth or inflation. Section 4 considers a money-capital economy in which taxes are the residual mode of government finance. A positive birth rate then raises the national income share of private consumption, depresses real growth, and increases inflation. Monetary growth is no longer neutral because it boosts real growth and therefore inflation rises by less than monetary growth. Balanced-budget increases in government consumption are shown to cut real growth and boost inflation. Section 5 analyzes what happens when monetary growth is the residual model of government finance. Section 6 allows for changes in public debt and shows that postponing taxes reduces growth and increases inflation. Bond-financed changes in government consumption are shown to reduce growth more than tax-financed changes whilst money-financed changes reduce growth less. Similarly, when open market operations are used to raise monetary growth, real growth rises by more and inflation by less than when subsidies are used to raise monetary growth. Section 7 briefly examines the case of nonseparable preferences. Section 8 introduces costs of adjustment for investment. This makes the real interest rate endogenous and thus introduces an additional channel through which money and government debt can affect real growth and inflation. Section 9 concludes with a summary of results.

\textsuperscript{5} Clearly, fiscal policy is non-neutral (even within the context in which Ricardian debt neutrality holds) when financed by distortionary taxes. For example, an increase in public consumption financed by a tax on output or on capital depresses growth.

\textsuperscript{6} Blackburn and Hung (1991) focus on the effect of productive government spending while Jones and Manuelli (1991) and Mino (1991) focus on the effect of human capital formation on endogenous growth.

\textsuperscript{7} Jones and Manuelli (1991) pay attention to the related effects arising from nominally denominated depreciation allowances in the tax code.
2. OVERLAPPING GENERATIONS, MONEY AND ENDOGENOUS GROWTH

2.1. Household Behavior and Aggregation

Departures from debt neutrality (Barro 1974) are ensured in an economy of overlapping generations, as long as the birth rate, $\beta$, is positive (Weil 1989). There is a constant probability of death, $\lambda$, so population growth is defined by $n = \beta - \lambda$. The uncertain-lifetimes approach of Blanchard (1985) corresponds to no population growth ($n = 0$) so that the birth rate equals the death rate ($\beta = \lambda$). Finite lifetimes are thus not necessary to depart from debt neutrality.

The representative household of the cohort born at time $s$ solves at time $t$:

$$\max \int_0^\infty \exp[-(\rho + \lambda)](v - t)\log(\Omega(\tilde{c}(s, v), \tilde{m}(s, v)))dv$$ (1)

subject to the household’s instantaneous flow identity

$$\frac{d\bar{a}(s, t)}{dt} = [r(t) + \lambda]\bar{a}(s, t) - \bar{j}(s, t) - \bar{\tau}(s, t) - \bar{c}(s, t)$$

$$- [r(t) + p(t)]\tilde{m}(s, t)$$ (2)

and the household’s solvency condition

$$\lim_{v \to \infty} \exp\left(-\int_r^v [r(w) + \lambda]dw\right)\bar{a}(s, v) = 0$$ (3)

where $\rho$ stands for the pure rate of time preference, $r(t)$ and $p(t)$ denote the real interest rate and the inflation rate at time $t$, and $\tilde{c}(s, t)$, $\tilde{m}(s, t)$, $\tilde{a}(s, t)$, $\tilde{j}(s, t)$, and $\tilde{\tau}(s, t)$ denote at time $t$ the levels of consumption, real money balances, nonhuman wealth, nonasset income and lump-sum taxes (net of transfers) of a household born at time $s$, respectively.

Equation (1) says that the household maximizes expected utility, where the discount rate is the sum of the death probability and the pure rate of time preference. Comprehensive consumption ($\bar{x}$) consists of consumption of goods plus interest foregone on money holdings:

$$\bar{x}(s, t) = \tilde{c}(x, t) + [r(t) + p(t)]\tilde{m}(s, t).$$ (4)

$\Omega(.)$ is homothetic and homogenous of degree one. Equation (2) says that the return on assets, that is, interest plus the life insurance premium, plus nonasset income, minus taxes, must be consumed or saved. When the household is alive it receives a premium of $\lambda \tilde{a}$ on its assets and when it dies the estate goes to the insurance company, hence the premium is actuarially fair. Free entry and exit ensure that the insurance industry makes no profits. Equation (3) says that households cannot play Ponzi
games. At stage 1 the household decides on comprehensive consumption, which yields the familiar tilt

\[ \frac{d\xi(s, t)}{dt} = [r(t) - \rho] \xi(s, t) \]  

and thus from the household’s present value budget constraint

\[ \xi(s, t) = (\rho + \lambda) \left[ a(s, t) + \int_0^\infty [\tilde{j}(s, v) - \tilde{\tau}(s, v)] \exp \left( - \int_t^v [r(w) + \lambda] dw \right) dv \right]. \]

Comprehensive consumption is linear in total wealth, because (1) implies a constant elasticity of intertemporal substitution. In fact, (1) corresponds to a unit-elastic intertemporal utility function so that the propensity to consume out of wealth does not depend on the real interest rate. The propensity to consume increases with the probability of death.

The fraction of the cohort born at time \( s \) still alive at time \( t \) equals \( L(s) \exp(-A(t-s)) \), where \( L(s) \equiv \exp(ns) \) is the population size at time \( s \). Population aggregates are defined as

\[ X(t) = \beta \exp(-\lambda t) \int_{-\infty}^t \xi(s, t) \exp(\beta s) ds. \]

Aggregation over cohorts, assuming that individuals do not inherit any wealth, \( a(s, s) = 0 \), yields

\[ \dot{X}(t) = [r(t) - \rho + n]X(t) - \beta(\lambda + \rho)A(t) ; \]  

\[ \dot{A}(t) = r(t)A(t) + J(t) - T(t) - X(t) . \]

At stage 2 the household optimizes on the allocation between consumption of goods and real money balances. This requires that the marginal rate of substitution between goods and real money balances equals the opportunity cost of holding real money balances (that is, the nominal interest rate), \( \Omega_m/\Omega_c = r + p \), so that \( \tilde{c}(s, t) = \Gamma[r(t) + p(t)]\tilde{m}(s, t) \), where \( \Gamma' > 0 \). Upon substitution into (4) and aggregation, one obtains

\[ M = \Psi(r + p)X, \quad \Psi' = -(1 + \Gamma')\Psi^2 < 0 ; \]  

\[ C = \Phi(r + p)X, \quad \Phi' = [(r + p)\Gamma' - \Gamma]\Psi^2 ; \]

\[ q = [\Psi(r + p)\Omega(\Gamma(r + p),1)]^{-1} . \]
Homothetic $\Omega(.)$ implies that goods and real money balances are normal goods. A CES utility function $\Omega = [\gamma \xi + (1 - \gamma)\bar{m}\xi]^{1/\xi}$, $\xi \neq 1$, $0 < \gamma < 1$, yields $\Gamma = \gamma(r + p)/(1 - \gamma)\Psi^{-1}$, $\Psi = (1 - \gamma)[(1 - \gamma)(r + p) + \gamma(r + p)^{\sigma}]^{-1}$ and $\Phi = [1 + (\gamma^{-1} - 1)\sigma(r + p)]^{1-\sigma}$. Cobb-Douglas subpreferences ($\sigma = 1$) implies $\psi = \gamma \sigma = \gamma \sigma_{c.m}$. Cobb-Douglas subpreferences ($\sigma = 1$) implies $\psi = \gamma \sigma = \gamma \sigma_{c.m}$. Cobb-Douglas subpreferences ($\sigma = 1$) implies $\psi = \gamma \sigma = \gamma \sigma_{c.m}$.

In general, the budget share of goods falls (rises) with the nominal interest rate when the income (substitution) effect dominates, that is, when $\sigma < (>) 1$.

### 2.2. Behavior of Firms

Following the recent literature on endogenous growth, we assume constant returns to scale with respect to a broad measure of the capital stock. We focus attention on demand-side rather than on supply-side policies so that the effects of productive government spending on the productivity of capital are ignored. The production function is of the Cobb-Douglas variety:

$$Y(t) = \theta K(t)^{\eta}[\bar{K}(t)/\bar{L}(t)L(t)]^{1-\eta}, \quad 0 < \eta \leq 1$$ (12)

where $Y(t)$, $L(t)$, $\bar{L}(t)$, $K(t)$, and $\bar{K}(t)$ denote the level of production, the own level of employment, the aggregate level of employment, the own capital stock, and the aggregate capital stock at time $t$, respectively. Production exhibits constant returns to scale with respect to $K$ and $\bar{K}$. If $\eta$ is less than unity, there is learning by doing because then knowledge from one producer spills over and increases the output of rival firms. Capital accumulation satisfies $\bar{K}(t) = I(t) - \delta K(t)$ where $I(t)$ denotes gross investment at time $t$ and $\delta$ denotes the depreciation rate. Producers borrow and lend at a given interest rate and maximize the present value of their net revenues. In the absence of adjustment costs, the marginal productivity of capital must equal the rental plus the depreciation charge. The marginal productivity of labor must equal the real wage. In symmetric equilibrium one has $Y = \theta K$ and $r = \eta \theta - \delta$. Learning by doing implies that the private rate of return on assets is less than the social rate of return. Since $(r + \delta)K = \eta Y \leq Y$, national income exceeds the income from private capital. Learning by doing and knowledge spillovers provide wage income to households, irrespective of their age, so that $J = (1 - \eta)Y$ corresponds to the wage bill.

### 2.3. Government

Public consumption ($G$) plus interest on the public debt ($rD$) must be financed by taxes ($T$), borrowing ($\dot{D}$) or printing money ($\mu M$). The flow budget identity of the government is

$$\dot{D}(t) = r(t)D(t) + G(t) - T(t) - \mu(t)M(t)$$ (13)
where $D(t)$, $\mu(t)$ and $\mu(t)M(t)$ denote public debt, growth in the nominal money supply, and seigniorage revenues at time $t$, respectively. Solvency of the public sector implies

$$D(t) + \int_t^\infty \exp \left[ - \int_t^\nu r(w)dw \right] G(v)dv \leq \int_t^\infty \exp \left[ - \int_t^\nu r(w)dw \right] [T(v) + \mu(v)M(v)]dv. \quad (14)$$

Current government debt plus the present value of future government consumption must not exceed the present value of taxes plus seigniorage revenues. The national income share of government consumption, $g \equiv G/Y$, is exogenous. Section 4 refers to tax finance and considers a pure money-capital economy without government debt in which taxes are the residual instrument, that is, $T = G - \mu M$, and $g$ and $\mu$ are exogenous. Section 5 refers to money finance and considers a money-capital economy in which monetary growth is the residual instrument, that is, $\mu = (G - T)/M$. Section 6 analyzes bond finance and intertemporal shifts in taxation whilst keeping $g$ and $\mu$ exogenous. To ensure solvency, it is necessary to specify a tax rule which arrests the explosion of government debt:

$$T = \tau_0 Y + \tau_1 D. \quad (15)$$

The tax-GDP ratio rises with the government debt-GDP ratio. Solvency of the public sector requires that $\tau_1$ exceeds the growth-corrected real interest rate.

### 2.4. Equilibrium

Equilibrium in the product and money markets requires $Y = C + I + G$ and $\dot{M} = (\mu - p)M$. Equilibrium on the equity and bond markets implies $A = K + M + D$. Labor demand adjusts to employ the supply of labor (a constant fraction of the population).

The real growth rate of national income and capital is denoted by $\omega \equiv \dot{Y}/Y = \dot{K}/K$ while the per capita growth rate (or the endogenous rate of labor-augmenting technical progress) is defined as $\pi \equiv \omega - n$. It is convenient to express all variables in fractions of the national income, which are denoted by lower-case letters (for example, $c \equiv C/Y$). The model can be summarized by

$$\dot{c} = (\eta \theta - \delta - \rho - \pi)c$$

$$- (n + \lambda)(\lambda + \rho)\Phi(\eta \theta - \delta + \rho) \left( \frac{1}{\theta} + m + d \right); \quad (16)$$

$$\dot{m} = (\mu + \eta \theta - \delta - \pi - n)m - [\Phi(\eta \theta - \delta + p)^{-1} - 1]c; \quad (17)$$

$$\dot{d} = (\eta \theta - \delta - \pi - n - \tau_1)d + g - \tau_0 - \mu m \quad (18)$$
where the per capita real growth rate is given by

\[ \pi = \theta (1 - c - g) - \delta - n \]  

(19)

and the rate of inflation by

\[ p = \Gamma^{-1}(c/m) - \eta \theta + \delta . \]  

(20)

Equation (19) is the condition for equilibrium in the goods market. It says that the growth rate equals the propensity to save divided by the capital-output ratio, minus the depreciation rate. This condition therefore corresponds to the Harrod-Domar (HD) rule familiar from the older literature on economic growth.

Sections 3–5 restrict attention, for simplicity, to the case of Cobb-Douglas subutility functions (\( \sigma = 1 \)), so that \( \Phi(r + p) = \gamma \). Section 6 then briefly examines the more general class of CES subutility functions.

3. RICARDIAN DEBT NEUTRALITY AND ENDOGENOUS GROWTH

Ricardian debt neutrality holds when there is no entry of new generations of households and the birth rate is zero (\( \beta = n + \lambda = 0 \)). Figure 1 demonstrates how the national income share of private consumption and per capita growth are determined. The HD locus slopes downward because a higher national income share of private consumption implies a lower share of investment and thus a lower growth rate of the capital stock. Equation (16) reduces to the Ramsey-Romer (RR) rule if the birth rate is zero, that is, the growth rate of per capita private consumption equals the difference between the market interest rate, given by the productivity of capital minus the rate of depreciation, and the pure rate of time preference (\( \pi = \pi^* = \eta \theta - \delta - \rho \)). A high interest rate induces households to save and postpone consumption, and thus yields a high growth rate. Because the rate of growth does not depend on the national income share of private consumption, as long as
debt neutrality holds, the RR rule is vertical. The equilibrium per capita growth rate of the economy (\(\pi = \pi^*\)) thus depends only on economic policy in as far as the government is able to influence, say, through investment in infrastructure, the marginal productivity of capital. Per capita growth does not depend on intertemporal shifts in taxation and government debt, on the national income share of government consumption, or on the monetary growth rate. It does decrease as households become more impatient and as the degree of learning by doing increases.

The equilibrium national income share of private consumption (\(c = c^* \equiv ((\rho - n)\theta^{-1} + 1 - \eta - g)\)) increases with the pure rate of time preference and the degree of learning by doing but decreases with population growth and the national income share of government consumption (move from E to E'). In fact, an increase in government consumption leads to 100 percent crowding out of private consumption so that private investment and thus the growth rate of the economy are unaffected. However, the national income share of private consumption does not depend on intertemporal shifts in taxation, government debt, or on monetary growth. These policy neutrality results hold for the general class of homothetic subutility functions.

Since debt neutrality implies a dichotomy between the real and monetary sides of the economy, it is possible to obtain from equations (17) and (20)

\[
\dot{p} = (\eta \theta - \delta + p) (\omega + p - \mu), \quad \omega = \eta \theta - \delta - \rho + n.
\]

The perfect-foresight solution of this Bernoulli equation is given by

\[
p(t) = \left[ \int_t^\infty \exp \left( - \int_t^\nu [\rho - n + \mu^c(w, t)] dw \right) dv \right]^{-1} - \eta \theta + \delta,
\]

where \(\mu^c(w,t)\) denotes the expectation of \(\mu(w)\) conditional on information available at time \(t\). Hence, equilibrium inflation depends on an average of all future monetary growth rates and does not depend on budgetary demand-side policies whatsoever. Inflation increases when households become more impatient, when population growth falls, when the marginal productivity of capital falls, when the degree of learning by doing increases, and when the depreciation rate increases, because in all these cases the real growth rate of the economy falls. Steady-state inflation equals the excess of monetary growth over growth in the real national product, \(p = \mu - \omega\). Hence, money is superneutral as long as debt neutrality holds. The remainder of the paper focuses attention on economies in which new noninterconnected generations of households enter the economy and as a result debt neutrality does not hold.

4. FINANCE BY LUMP-SUM TAXATION

This section considers an economy with a positive birth rate, a constant government debt-GDP ratio, and lump-sum taxation as the residual mode of government finance. Substitution of (19) into (16) and (17) yields
\[
\dot{c} = \theta c^2 + [\theta g + n - \rho - (1 - \eta)\theta]c - (n + \lambda)(\lambda + \rho)\gamma \left( \frac{1}{\theta} + m + d \right); \quad (16')
\]

\[
\dot{m} = \theta cm + [\theta(g + \eta -1) + \mu]m - \left( \frac{1 - \gamma}{\gamma} \right) c. \quad (17')
\]

Figure 2 presents the phase diagram associated with (16')–(17'). The steady-c locus reduces to the Ramsey-Romer rule, \( c = c^* \equiv (\rho - n)\theta^{-1} + 1 - \eta - g \), when the birth rate is zero, which is as discussed in section 3 independent of real money balances. In general, the birth rate is positive and the steady-c locus slopes upward in the quadrant where both \( c \) and \( m \) are positive, as an increase in \( m \) means that households have more wealth and thus consume more. The steady-c locus is a quadratic and achieves a minimum value of \( m \) when \( c = \frac{1}{2}c^* \). Because the steady-c locus cuts both the RR rule and the horizontal axis in the region where \( m \) is negative, that is, when \( m = -\theta^{-1} - d \), the steady-c locus slopes upward in the positive quadrant. Because households anticipate the entry of new generations who help to carry the burden of future taxes, they consume more for a given level of real money balances than they would have done otherwise. Consequently, the steady-c locus is in the relevant quadrant above the RR rule.

The steady-m locus goes through the origin, slopes upward, and has an asymptote, given by \( m = (1 - \gamma)(\gamma\theta)^{-1} \). There are two forces at work. The first is that an increase in private consumption must, for a given nominal interest rate, be associated with an increase in holdings of real money balances. The second is that an increase in private consumption leaves fewer resources for investment, so that real growth falls, inflation and the nominal interest rate rise, and consequently holdings of real money balances fall. The first force ensures that the steady-m locus slopes upward, whilst the second force ensures that it does not cross the asymptote.

With an unanticipated permanent shock, holdings of real money balances and consumption immediately jump to their new equilibrium levels. There are no transi-
tional dynamics because both \( c \) and \( m \) are nonpredetermined. It is instructive to also consider the determination of the equilibrium levels of inflation and real per capita growth. This is illustrated in Figure 3. The points describing equilibrium in the money market must lie on the MM locus, \( p = \mu - n - \pi \), which says that growth in nominal national income (that is, per capita real growth plus population growth plus inflation) must equal monetary growth. The points describing equilibrium in the goods market lie on the GG locus:

\[
c = 1 - g - \left( \frac{\delta + n + \pi}{\theta} \right)
\]

\[
= \left( \frac{(n + \lambda)(\lambda + \rho)\gamma(\theta^{-1} + d)}{\eta\theta - \delta - \rho - \pi - \left[ \frac{(n + \lambda)(\lambda + \rho)(1 - \gamma)}{\eta\theta - \delta + \rho} \right]} \right)
\]

which is obtained upon substitution of the (steady-state) consumption and money demand functions, (16')–(17'), into the Harrod-Domar rule, (19). In the absence of entry of new generations of households \( (n + \lambda = 0) \), debt neutrality hold. Hence, (22) reduces to the familiar Ramsey-Romer rule, and economic growth is independent of demand-side policies and inflation \( (\pi = \pi^* \equiv \eta\theta - \delta - \rho) \). Higher inflation leads, as the real interest rate is given by technology and supply-side policy, to a higher nominal interest rate, so depresses holdings of real money balances and consequently reduces private wealth and private consumption. If the birth rate \( (n + \lambda) \) is positive, more resources become available for investment purposes so that the real growth rate increases. This is why the GG locus slopes upward. It can be established from Figure 2 and equation (22) that the GG locus lies entirely to the left of the RR rule.

Absence of an intergenerational bequest motive combined with the birth of new generations induces households to allocate a greater proportion of income to private
consumption and to hold more real money balances. This means that a smaller share of income is devoted to investment and saving, so that real per capita growth is less, and consequently, inflation is higher for a given rate of growth in the nominal money supply than in a world in which debt neutrality holds (compare $E_1$ with $E_0$ in Figures 2 and 3).

If households become more patient (lower value of $\rho$), it is straightforward to establish that the steady-$c$ and the RR loci shift downward in Figure 2 so that the equilibrium national income share of private consumption and holdings of real money balances fall. As a result, there is more room for investment so that the real growth rate rises and inflation falls (the GG and RR loci in Figure 3 shift to the right). More patience reduces inflation, irrespective of whether there is a zero or a positive birth rate.

**4.1. Non-neutrality of Monetary Growth**

Consider an increase in monetary growth where the seigniorage revenues are given back to the private sector in the form of lump-sum transfers (independent of the age of the recipients). Such a “helicopter drop” increases inflation and nominal interest rates, so real money balances fall. Consequently, the steady-$m$ locus in Figure 2 and the MM locus in Figure 3 shift upward. In a Ricardian world the inflation and nominal interest rates rise by exactly the same amount as monetary growth because the national income share of private consumption and thus the share of investment and the real growth rate are unaffected (move from $E_0$ to $E_0'$).

However, when the benefits of future tax cuts or subsidies can no longer be fully enjoyed because households may no longer be alive or because they have to be shared with future generations, private consumption falls. As a result, more resources are available for investment, real growth is boosted and inflation rises by less than monetary growth (move from $E_1$ to $E_1'$). Increases in monetary growth are thus non-neutral in two senses: the real growth rate of the economy increases and inflation is not entirely a monetary phenomenon. The first type of non-neutrality is also found in economies with noninterconnected overlapping generations of households and decreasing returns to capital. These models, however, provide the micro foundations of the Tobin effect in the sense that they show that an increase in monetary growth depresses the real interest rate and thus boosts capital accumulation and the level of output (Marini and van der Ploeg 1988). Here, a sustained effect of monetary growth on real growth, rather than on the level of output, is found.

**4.2. Government Consumption Reduces Growth and Causes Inflation**

A balanced-budget increase in the national income share of government consumption depresses the share of private consumption and thus shifts the steady-$c$ locus and the RR locus in Figure 2 downward. It also shifts the steady-$m$ locus in Figure 2 upward. The GG locus shifts upward in Figure 3, but the RR locus in Figure 3 is unaffected. In an economy in which debt neutrality holds, there is thus 100 percent crowding out of private consumption so that the national income shares of
investment and saving, real growth, and inflation are unaffected. Real money balances fall by the same percentage as private consumption (move from $E_0$ to $E''_0$).

In an economy in which the burden of future taxes is shared with future generations ($n + \lambda > 0$), private consumption is not fully crowded out. As a result, fewer resources are available for private investment so real growth falls. Because the inflation and nominal interest rates rise, real money balances fall by a greater percentage than private consumption (move from $E_1$ to $E'_1$). A consistent model has thus been posited that suggests a direct positive association between balanced-budget changes in the national income share of government consumption and the inflation rate.\footnote{Similar results are obtained in a world in which Ricardian debt neutrality ($\beta = 0$) holds when one has a simple linear income tax with the proceeds distributed in a lump-sum fashion (cf. Barro 1990) instead of a lump-sum tax. A balanced-budget increase in the national income share of government consumption then also reduces growth and, with a passive monetary policy, raises inflation.}

However, if monetary growth changes with real growth (rather than being kept constant), an increase in the national income share of government consumption leaves inflation unaffected.

5. FINANCE BY PRINTING MONEY

Now assume that monetary growth is the residual mode of government finance and that the tax rate and the national income shares of government consumption and government debt are exogenous constants. Monetary growth must then generate sufficient seigniorage revenues to finance the primary deficit plus real debt service:

$$\mu m = \theta(c + g + \eta - 1)d + g - \tau .$$

Upon substitution of (23) into (17'), one obtains

$$\dot{m} = \theta cm + \theta(c + g + \eta - 1)d + \theta(g + \eta - 1)m + g$$

$$- \tau - \left(\frac{1 - \gamma}{\gamma}\right)c .$$

If one starts with no government debt and a zero primary deficit, monetary growth is zero and the steady-$m$ locus is the same irrespective of whether there is tax finance (TF) or money finance (MF). Figure 4 then shows the effects of an increase in the national income share of government consumption under these two modes of government finance. Since the expansion of monetary growth that is associated with the increase in government consumption also crowds out private consumption, it is clear that a money-financed increase in government consumption leads to more crowding out of private consumption, a smaller fall in investment, and thus a smaller fall in the real growth rate of the economy than a balanced-budget increase in government consumption (move from $E$ to $E''$ rather than to $E'$ in Figure 4).
ously, the price one has to pay for money finance is a higher increase in inflation than under tax finance. Note that, when Ricardian debt neutrality holds, neither a tax-financed nor a money-financed change in the national income share of government consumption affects real growth.

6. FINANCE BY ISSUING BONDS

6.1. Intertemporal Shifts in Taxation

Consider a cut in the current fraction of national income collected in taxes followed by gradual increases in future tax rates as the ratio of government debt to national income rises over time. The present value of increases in future taxes must exactly equal the cut in current taxes. The steady-state effects of such a postponement of taxes may be deduced from the effects of an increase in the ratio of government debt to national income on equations (16') and (17'). When there is no entry of new generations of households, debt neutrality prevails. The national income shares of private consumption and real money balances are unaffected as households realize that the current tax cut must be fully paid for by future taxes. As a result, real growth and inflation are unaffected. However, if there is a positive birth rate, a tax cut boosts private consumption as households realize that future taxes are also paid for by future generations (as can be seen from the effects of an upward shift of the steady-c locus in Figure 2). This leaves fewer resources for investment and saving, so that real growth falls and thus inflation rises (as can be seen from the effects of an upward shift of the GG locus in Figure 3). Hence, a transitory tax cut leads to a long-run fall in real growth and a permanent increase in inflation. It follows that future taxes must rise by more than in conventional theories of economic growth because the growth-corrected real debt service rises not only due to the increase in government debt but also due to the increase in the growth-corrected real interest
rate. The literature on the burden of government debt thus gains a new perspective once cast within the new theories of endogenous growth. Government debt hurts investment and thus growth, and boosts inflation, even though the real interest rate and monetary growth rate are unaffected.

It is straightforward to establish that the short-run effects of a transitory tax cut on the national income share of private consumption are larger than the long-run effects. Hence, in the short run real growth falls and inflation rises by more than in the long run. The development of the ratio of government debt to national income follows from

$$d = \theta cd - (\tau_1 - \theta g)d + g - \tau_0 - \mu m \ . \quad (18')$$

As over time the share of private consumption falls and real growth rises, the burden of real debt service falls and the process of arresting the growth in the government debt becomes easier.

6.2. Government Consumption

The effects of a bond-financed increase in government consumption may be deduced from adding the effects of a balanced-budget increase in government consumption to the effects of a gradual increase in government debt. The effects of the first shock are less than 100 percent crowding out of private consumption and thus a fall in real growth and an increase in inflation (see section 4.2). The effects of the second shock are an increase in private consumption, a fall in real growth, and an increase in inflation (see section 6.1). It thus follows that a bond-financed rise in government consumption leads to less crowding out of private consumption, a larger fall in real growth, and a larger increase in inflation than a tax-financed increase in government consumption. A money-financed increase in government consumption leads, however, to more crowding out of private consumption and thus to a smaller fall in real growth than a tax-financed increase in government consumption. Of course, if Ricardian debt neutrality holds, these three modes of government finance do not affect the national income shares of private consumption and investment or real growth, albeit that money finance induces an increase in inflation.

6.3. Open-Market Operations

Consider an increase in monetary growth implemented through open-market purchases of government bonds. The effects of an increase in monetary growth implemented through handing back lump-sum transfers to households is a reduction in the national income share of private consumption, an increase in real growth, and a less than 100 percent increase in inflation (see section 4.1). The long-run effects of buying back government debt and gradually cutting taxes are a fall in the national income share of private consumption, an increase in real growth, and a fall in inflation (see section 6.1). It follows that the use of open-market operations to raise monetary growth leads to a larger reduction in private consumption, a larger increase in real
growth, and a smaller increase in inflation than the use of subsidies. Note that the use of open-market purchases to raise monetary growth has no real effects in an equivalent model with diminishing rather than constant returns to capital at the aggregate level (Marini and van der Ploeg 1988).

7. NONSEPARABLE PREFERENCES

If one allows for the more general class of CES subutility function \( \sigma \neq 1 \), the nominal interest rate is given by \( (1 - \gamma)\gamma^{-1}(C/M)^{1/\sigma} \) and equations (16') and (17') become

\[
\dot{c} = \theta c^2 + \left[ \theta (g + n - \rho (1 - \eta)\theta) - (n + \lambda)\gamma \left( \frac{\theta^{-1} + m + d}{\gamma + (1 - \gamma) \left( \frac{c}{m} \right)^{(1-\sigma)/\sigma}} \right) \right];
\]

\[
m = \theta cm + \left[ \theta (g + \eta - 1) + \mu \right] m - \left( \frac{1 - \gamma}{\gamma} \right) \left( \frac{c}{m} \right)^{1/\sigma} m.
\]

If the birth rate is zero and debt neutrality holds, \( c = c^* \), \( \pi = \eta \theta - \delta - \rho \) and \( p = \mu - \eta \theta + \delta + \rho - \eta \) are as before. Long-run holdings of real money balances are now given by \( m = \left[ (1 - \gamma)/\gamma (\mu + \rho - n) \right] c^* \), so the negative effects of higher monetary growth on holdings of real money balances is greater if the elasticity of substitution between goods and real money balances is higher.

The extreme case of an infinite elasticity of substitution leads to an exogenous nominal interest rate, \( (1 - \gamma)/\gamma \), so that inflation, \( p = (1 - \gamma)\gamma^{-1} - \eta \theta + \delta \), is independent of monetary and/or fiscal policies. Per capita real growth, \( \pi = \mu - n - (1 - \gamma)\gamma^{-1} + \eta \theta - \delta \), then increases one-for-one with the rate of monetary growth. The national income share of private consumption, \( c = (1 - \gamma)(\gamma \theta)^{-1} - (\mu/\theta) + 1 - \eta - g \), decreases one-for-one with the national income share of public consumption, whether debt neutrality holds or not, and decreases with nominal money growth.

8. MONEY, GOVERNMENT DEBT, AND THE REAL INTEREST RATE

So far, we assumed a constant real interest rate. This is somewhat unrealistic because one expects money and government debt to affect the real interest rate. Also, in conventional models of economic growth with diminishing returns to capital and finite lives an increase in monetary growth induces a Tobin effect in the sense that the real interest rate is reduced and the capital-output ratio is increased (Marini and
van der Ploeg 1988). The easiest way to make the real interest rate endogenous in the new growth theories, apart from using supply-side policies to affect the marginal productivity of capital, is to introduce costs of adjustment for investment (Barro and Sala-i-Martin 1990; Alogoskoufis and van der Ploeg 1990, 1991).

The representative firm then maximizes the present value of net revenues:

$$\max \int_0^\infty \left[ \theta K(v)\bar{K}(v)^{1-\eta} - I(v) - \phi \left( \frac{I^2(v)}{K(v)} \right) \right] \exp \left[ -\int_t^v r(w)dw \right] dv$$

(24)

where $\phi$ denotes the cost-of-adjustment parameter. Costs of adjustment are proportional to the investment rate. The optimal investment program satisfies in symmetric equilibrium:

$$I = \frac{1}{2}(q - 1)r^4 - \lambda K; \quad (25)$$

$$q = \theta + \phi \left( \frac{I}{K} \right)^2$$

(26)

where $q$ denotes the value of capital to the firm (Tobin’s Q). Equation (25) requires that the marginal cost of adjusting the capital stock plus the purchase cost of investment goods must equal Tobin’s Q. Equation (26) says that the user cost of capital (that is, the rental charge plus depreciation charge minus capital gains) must equal the marginal productivity of capital plus the marginal reduction in adjustment costs arising from an additional unit of capital. Since $I/K = \pi + n + \delta$, one obtains from (25) and (26) the following long-run relationship:

$$r + \delta = [\eta \theta + \phi(\pi + n + \delta)^2][1 + 2\phi(\pi + n + \delta)]^{-1}.$$  

(27)

If costs of adjustment are zero, the real interest rate is constant ($r = \eta \theta - \delta$ if $\phi = 0$). If costs of adjustment are positive, (27) defines a negative relationship between the real interest rate and the real growth rate. The point is that a higher real interest rate reduces Tobin’s Q and thus reduces the investment rate and the real growth rate. Equation (27) is portrayed as the production locus in Figure 5. If $\sigma = 1$, growth in private consumption is given by

$$\pi = r - \rho - (n + \lambda)(\lambda + \rho)$$

$$\times \left[ \left( \frac{\gamma(\theta^{-1} + \delta + \pi + n)}{1 - g - (\delta + \pi + n)\theta^{-1}} \right) + \left( \frac{1 - \gamma}{r + \mu - \pi - n} \right) \right].$$

(28)

Equation (28) defines a positive relationship between the real interest rate and per capita growth because a high real interest rate induces households to postpone con-
consumption. Also, an increase in the real interest rate depresses human wealth and private consumption and thus boosts real growth. Equation (28) is portrayed as the consumption locus in Figure 5. Adjustment costs for investment imply in long-run equilibrium lower real growth and a lower real interest rate ($E_1$ rather than $E_0$).

An expansion of government debt shifts up the consumption locus, so leads to a fall in real growth and an increase in the real interest rate (move from $E_1$ to $E'_1$). It also induces an increase in inflation and nominal interest rates. A tax-financed increase in government consumption has similar effects on the real interest rate and real growth. An expansion of monetary growth has the opposite effects. It shifts down the consumption locus and leads to an increase in real growth and a fall in the real interest rate (which is the Mundell effect).

Most theories of endogenous growth assume Ricardian debt neutrality and consequently encounter an empirical puzzle. If preferences ($\rho$) are relatively stable and shifts in technology and more specifically the productivity of capital are common, the consumption locus is relatively fixed whilst the production locus moves about. This implies a positive correlation between real interest rates and growth rates, but empirically it is hard to detect such a correlation. Barro and Sala-i-Martin (1990) therefore extend their model and show that growth in the variety of consumer products is like a shift in the rate of time preference, so that a significant amount of technical progress involving types of consumer products relative to technical change involving varieties of capital goods may explain the lack of correlation between real interest rates and growth rates. The present analysis suggests a much more straightforward explanation: changes in budgetary policies move about the consumption locus and suggest, if anything, a negative correlation between real interest rates and growth rates. The analysis also gives insight into another empirical puzzle, namely, the observed negative correlation between inflation and growth (for example, Fischer 1991).

9. CONCLUDING REMARKS

As long as Ricardian debt neutrality holds, monetary growth does not affect the national income shares of private consumption and investment or real growth and is
thus the sole determinant of inflation. Also, government consumption leads to 100 percent crowding out of private consumption and thus does not affect the real growth and inflation rates either. In order to have an interesting analysis of macroeconomic policy issues, it is crucial to depart from debt neutrality. This is achieved when there is no operational bequest motive and entry of future generations of households because then the burden of future taxes is shared with new generations. As a result, for a given stance of monetary and fiscal policy, the national income share of private consumption is higher and consequently real growth is less and inflation is higher than in an economy populated by dynasties with an operational bequest motive. An increase in monetary growth is thus not superneutral and, in addition, leads to a less than 100 percent increase in inflation. If the increase in monetary growth is accompanied by open-market purchases of bonds rather than lump-sum subsidies, there is a larger increase in real growth and a smaller increase in inflation, so money is even less neutral. A balanced-budget increase in the national income share of government consumption leads to less than 100 percent crowding out of private consumption, a fall in real growth, and an increase in inflation. A money-financed increase in government consumption leads to less crowding out and thus to a smaller fall in real growth and a bigger increase in inflation than a tax-financed increase. On the other hand, a bond-financed increase in government consumption leads to a bigger fall in real growth and a smaller increase in inflation than a tax-financed increase in government consumption. In general, an increase in government debt, arising from an intertemporal shift in taxation, reduces real growth and boosts inflation. If there are costs of adjustment for investment, increases in monetary growth lower real interest rates (Mundell effect) whilst increases in the government debt–GDP ratio raise real interest rates. Once the new theories of endogenous growth are cast within a framework of noninterconnected overlapping generations, the analysis of macroeconomic policies and the debates on the burden of government debt and the causes of inflation gain an exciting new perspective.

LITERATURE CITED


